14.7/8 Applied Max/Min

Method

Step 1: Draw and label

Step 2: Want? (objective) $z = f(x, y)$

Step 3: Given? (constraint) $g(x, y) = c$

Step 4: Engage calculus muscles

Step 5: Make conclusions.

Entry Task: *Identify objective and constraint.*

1. Find the points closest to $(6,2,0)$ on the cone $z^2 = x^2 + y^2$.

2. Find the largest box volume in the first octant with a vertex on $x + 2y + 3z = 3$

3. An open-top cardboard box has volume 27,436 cm³. Minimize material.

4. An open-top rectangular aquarium must hold 100 ft^3 of water. The bottom and back are slate costing \$10 per ft^2. The other sides are glass costing \$2 ft^2. Find the dimensions that minimize the total cost.

Optimization Techniques

Method 1: Substitution

Use the constraint to eliminate a variable in the objective. Find critical points of the resulting function. Check endpoints and boundaries. Compare output values.

Method 2: Lagrange Multipliers

Find extrema of f(x,y,z) subject to g(x,y,z) = c.

$$\nabla f = \lambda \nabla g$$

$$g(x, y, z) = c$$

Check special cases, endpoints, and boundaries. Compare output values.

Minimum Distance

Example 3 — Closest Points *(from last lecture)*

Find the points on the parabola $y = x^2$ closest to (0, 2).

Minimum Cost

Homework Q5 — Aquarium

An open-top aquarium has volume 100 in^2, a slate base, and glass sides. Slate costs seven times as much per unit area as glass.

Find the dimensions minimizing material cost.

Choose your own lecture…

What example do you want to do next from the handout or homework?